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Nonlinear steady-state heat conduction model for the Indian crust

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Abstract

Temperature is an important parameter that controls various tectonic processes. Therefore, estimation of the thermal structure of crust/ lithosphere is required for understanding a variety of geological, geophysical and geochemical problems. Characterization of thermal structure for thermally equilibrated continental crust requires reliable models of radiogenic heat sources distribution and thermal conductivity, a parameter that determines how fast the Earth looses its internal heat. Geophysical mapping of the structure of Indian continental crust and measurement of radiogenic heat sources concentration for various crustal rocks have enabled development of realistic models of radiogenic heat sources distribution. Thermal conductivity can be determined either by using ab-initio calculations of quantum mechanics or by measuring it experimentally. Recent results of laboratory measurements have provided a reliable working model of thermal conductivity for crust-forming minerals and rocks. In this model, thermal conductivity depends on temperature, thus making the governing heat conduction equation non-linear. We have used this model of thermal conductivity and available heat flow and radiogenic heat generation data to obtain the depth distribution of the temperature for some provinces of the Indian shield.

Introduction

Steady-state heat conduction model of the Indian crust can be constructed by using models of thermal conductivity and radiogenic heat generation. Thermal conductivity of rocks depends on the temperature which makes the heat conduction equation nonlinear. Singh and Negi (1982) modeled the steady-state heat conduction for a simple form of temperature dependence of the thermal conductivity representing phonon contribution. Singh (1981), Bhattacharji and Singh (1984) and Manglik and Singh (1992) modeled steady-state heat conduction using both phonon and photon contributions to the thermal conductivity. In all these models, the depth distribution of radiogenic heat was taken as exponential which satisfied the linear heat flow and heat generation relationship. However, now this relationship is not the only way to get the depth distribution of radiogenic heat generation in the crust. Geophysical studies have provided the composition and structure of the crust and the depth distribution of radiogenic heat can be inferred from this knowledge. For the Indian crust, recent studies by Roy and Rao (2000), Kumar and Reddy (2004), Kumar *et al.* (2007a,b) and Manglik (2006) have given the depth distribution of the radiogenic heat sources at several locations in the Indian shield.

In a recent study, Whittington *et al.* (2009) used a more complex temperature dependence of thermal conductivity in the continental crust by obtaining empirical relationships for thermal diffusivity (κ) and specific heat (C_P) based on experimental results. In this study, the following relationships for thermal diffusivity and specific heat, respectively, have been used:

$$\begin{aligned} \kappa(T<846K) &= 567.3/T-0.062 \\ \kappa(T>846K) &= 0.732-0.000135T \end{aligned} \quad , \qquad (1)$$

$$\begin{aligned} C_P(T<846K) &= 199.50+0.0857T-5.0\times10^{-6}T^{-2} \\ C_P(T>846K) &= 229.32+0.0323T-47.9\times10^{-6}T^{-2} \end{aligned} \quad , \qquad (2)$$

where κ , T and C_P are measured in mm^2/s, K, and $J/mol.K$, respectively and the average molar mass is 221.78 g/mol. In the present study, we have used the above empirical relationships and also considered the density (ρ) as a function of temperature to obtain thermal conductivity using

$$K=\kappa\rho C_P\,, \qquad (3)$$

and have obtained depth distribution of the temperature for some provinces of the Indian shield. We first describe the nonlinear heat conduction theory and then show the results.

Nonlinear steady heat conduction theory

In the steady heat conduction state, the heat flux Q is governed by the following energy conservation law:

$$\nabla.Q+A=0\,, \qquad (4)$$

Where A is radiogenic heat in the crust. To get the equation for temperature T we need the following constitutive relationship between Q and T:

$$Q=-K\nabla T\,. \qquad (5)$$

Here K is the thermal conductivity. Negative sign indicates that the heat flows from high temperature to low temperature. Combining eq.(4) and (5), we get the equation governing temperature

$$\nabla.(K\nabla T)=-A\,. \qquad (6)$$

In case the horizontal variation in the steady temperatures can be ignored, we get equation for the temperature as:

$$\frac{d}{dz}\left(K\frac{dT}{dz}\right)=-A\,. \qquad (7)$$

To apply this equation to the Indian crust we need two boundary conditions. These are provided by prescribing surface temperature and heat flux which are observed quantities:

$$T(z=0)=T_S, \tag{8}$$

$$Q(z=0)=-q_S. \tag{9}$$

Further, we need the models of thermal conductivity and radiogenic heat.

Thermal conductivity model of the crust

Recently, a comprehensive model of heat diffusivity has been developed using careful experimental studies. This is given by (Whittington *et al.,* 2009):

$$\kappa=\left(\frac{d}{T}-e\right), \tag{10}$$

Where d,e are experimentally derived constants. Using the expressions for temperature dependence of density and specific heat as (Whittington *et al.,* 2009):

$$\rho=\rho_0\{1-\alpha(T-T_S)\}, \tag{11}$$

$$C_P=\left(a+bT-cT^{-2}\right), \tag{12}$$

We get the expression for thermal conductivity as

$$K(T)=AT^2+BT+C+DT^{-1}+ET^{-2}+FT^{-3}, \tag{13}$$

where,

$$A=rb\,;\quad B=qb+ra\,;\quad C=pb+qa\,;\quad D=pa-rc\,;\quad E=-qc\,;\quad F=-pc\,;$$

$$q=-(ge+dh);\quad r=eh\,;\quad g=\rho_0(1+\alpha T_S);\quad h=\rho_0\alpha\,, \tag{14}$$

and a,b,c are experimentally derived constants for specific heat, and ρ_0,α,T_S are reference density, coefficient of thermal expansion and reference temperature, respectively. The values of these parameters are given in Table-1.

Table-1. Values of various control parameters. Values of a,b,c,d,e are taken for $(T < 846K)$ (Whittington *et al.,* 2009).

Parameter	Value	Unit
a	8.9954E+02	$Wm^{-1}K^{-1}$
b	0.3864	$Wm^{-1}K^{-2}$
c	2.2545E+07	$Wm^{-1}K$
d	5.673E-04	$m^2s^{-1}K$
e	6.2E-08	m^2s^{-1}
ρ_0	2700	$kg.m^{-3}$
α	3.0E-05	K^{-1}
T_S	273.15	K

Radiogenic model of Indian crust

Rao *et al.* (1976) constructed linear heat flow and heat generation relationship for the northern and southern part of the Indian shield based on then available six data. Gupta *et al.* (1991) also constructed such a relationship for the southern part of the Indian shield. This linear relationship has been used to get depth distribution of radiogenic heat sources. In the northern part the thickness of the layer containing radiogenic heat sources came out to be 14.8 km whereas in the southern part of the shield it is 11.5 km (Gupta *et al.*, 1991) in contrast to the value of 7.5 km given by Rao *et al.* (1976). These values have been used by several workers to estimate thermal structure of the Indian shield. Recently, heat generation values in various rock units of the Indian shield have been measured and based on the crustal lithology depth distribution of radiogenic heat has been constructed by Roy and Rao (2000, 2003), Kumar and Reddy (2004), Kumar *et al.* (2007a,b), and Manglik (2006). We shall be using these data to calculate the nonlinear heat conduction model of the Indian crust.

Calculations of Crustal Temperatures

We shall be using eq. (7) with the thermal conductivity model given by eq. (13). We first transform eq. (7) in linear form in terms of another variable G, having dimension of temperature, defined as (Carslaw and Jaeger, 1959)

$$G = \frac{1}{K(T_s)} \int_{T_s}^{T} K(y)dy \quad . \tag{15}$$

We then get

$$\frac{d^2G}{dZ^2} = -\frac{1}{K(T_s)} A(Z) \, . \tag{16}$$

This is linear equation in G. The solution of eq. (16), using the boundary conditions given by eqs. (8) and (9), is:

$$G(z) = \frac{q_s}{K(T_s)} z - \frac{1}{K(T_s)} \int_0^Z (Z - y) A(y) dy \,. \tag{17}$$

Eq. (17) can be solved for an n-layered model of the crust, each layer having the radiogenic heat generation of $A_i \text{ for } z_{i-1} \le z < z_i \ (i = 1, \cdots, n)$, where z_i is the depth to the base of the i^{th} layer and $z_0 = 0$ represents the surface of the earth. The solution of eq. (17) is:

$$G_l(z) = \frac{1}{K(T_S)} \left[q_S z - \sum_{i=1}^{l-1} A_i \int_{z_{i-1}}^{z_i} (z - y) dy - A_l \int_{z_{l-1}}^{z} (z - y) dy \right], \tag{18}$$

where the depth point z lies in the l^{th} layer i.e. $z_{l-1} \le z < z_l$. Once $G(z)$ is known, $T(z)$ can be obtained by substituting eq.(13) in eq.(15). The resulting expression is:

$$G(z) = \frac{1}{K(T_S)} \left[\frac{A}{3}\left(T^3 - T_S^3\right) + \frac{B}{2}\left(T^2 - T_S^2\right) + C\left(T - T_S\right) + D \ln\left(\frac{T}{T_S}\right) - E\left(T^{-1} - T_S^{-1}\right) - \frac{F}{2}\left(T^{-2} - T_S^{-2}\right) \right] \tag{19}$$

This is a nonlinear equation in T. We have solved this equation by using Newton-Raphson method.

Thermal structure of some provinces of the Indian Shield

In the present study, we have considered four different regions of the Indian shield (Fig.1) namely Western Dharwar Craton (WDC), Eastern Dharwar Craton (EDC), northern Eastern Ghat Belt (NEGB), and southern Eastern Ghat Belt (SEGB), for which crustal models of averaged radiogenic heat generation have been proposed by integrating radioelement measurement in rocks with the crustal structures obtained by deep crustal seismic studies (Kumar *et al.*, 2007a). We assume the above models of radiogenic heat generation in the present analysis (Fig. 2) but use the thermal conductivity as described by eq.(13). The surface heat flow is lowest in WDC and it is highest for NEGB. However, the reduced heat flow is comparable at 14-16 mW/m^2 for EDC, NEGB, and SEGB whereas it is about 8 mW/m^2 for WDC

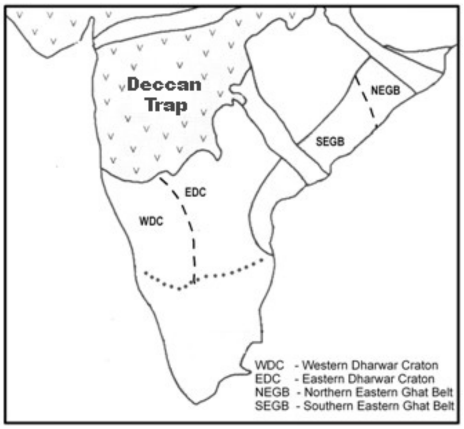

Fig. 1: A sketch map of the Indian shield showing WDC, EDC, NEGB, SEGB (modified after Kumar *et al.*,2007a)

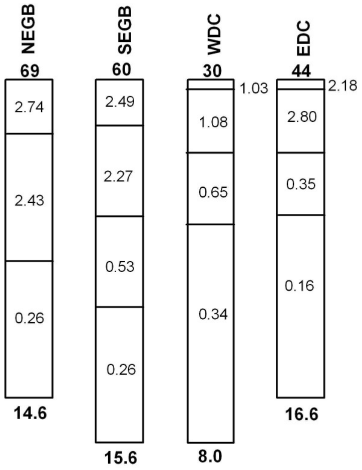

Fig. 2: Models of crustal structure along with values of radiogenic heat sources concentration used to calculate temperature-depth profiles. NEGB- Northern Eastern Ghat Belt, SEGB- Southern Eastern Ghat Belt, WDC- Western Dharwar Craton, EDC- Eastern Dharwar Craton (Kumar *et al.,* 2007a).

The results of depth distribution of temperature for all the four regions are shown in Fig. 3(a). The temperature profiles are plotted up to the Moho depth. For the present model of thermal conductivity, we obtain the Moho temperatures of 186°C, 229°C, 358°C, and 356°C for WDC, EDC, SEGB, and NEGB, respectively. These temperatures are lower than the one obtained by Kumar *et al.* (2007a) because a much smaller value of the thermal conductivity (2.0 – 2.5 mW/m^2) was used in those models. The empirical model of thermal conductivity (Whittington *et al.*, 2009) used in the present analysis yields a surface thermal conductivity of 3.8 $W/m.K$ which decreases with depth due to the increase in temperature. Depth variation of thermal conductivity for all four regions is shown in Fig. 3(b). A high value of thermal conductivity results in faster transport of heat in the crust and hence low temperatures.

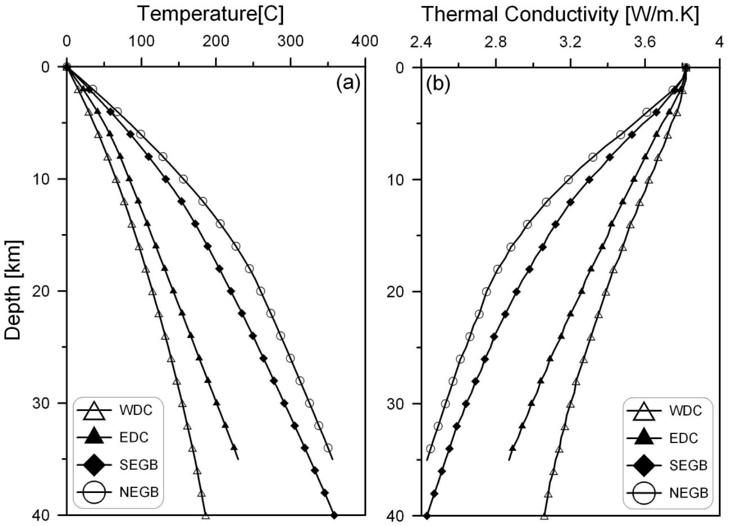

Fig. 3: (a) Temperature-depth profiles for the regions of the Indian shield shown in Fig 2, and (b) corresponding variation in thermal conductivity with depth.

In the present study, we have used an experimentally derived average model of thermal conductivity for the Indian region (Whittington *et al.*, 2009). A more suitable model of thermal conductivity specific to the Indian region should take in to consideration the high pressure-temperature measurements of rocks constituting the Indian crust.

Recently, attempts have been made to infer temperature in the crust from the shear wave velocity. The variations of temperature can also be inferred from electrical conductivity information which is being obtained by electrical and electromagnetic methods. These methods provide independent constraints on the thermal structure of the Indian crust and can help to constrain the models of thermal conductivity. Thus, integration of heat flow and heat generation measurements, high pressure-temperature experiments on crust forming minerals/ rocks, and seismological and electromagnetic observations can provide a better understanding of the thermal structure of the Indian crust.

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