

Stress response around fracture surface under uniaxial loading

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Abstract

Rock mass are generally considered to be the most heterogeneous, complex and unpredictable geomaterials. It is difficult to fully understand its behavior under various loading conditions. In the present paper, an attempt has been made to calculate the maximum stresses at the crack tips (σ_m) after fracture developed in a rock specimen. Rock specimen was subjected to loading on servo-controlled auto feedback stiff testing system under compressional mode in a Brazilian cage. The fractured rock surface was considered for the determination of stresses all along the crack tips. The specimens were prepared and tested as per International Society of Rock Mechanics specifications for tensile strength determination.

The specimen was placed under the Brazilian cage and subjected to compressional loading till it develop failure. The fracture mode followed the Griffith failure criteria. Various researchers have tried to calculate the stress at various point along the fracture plane and established relation with other physico-mechanical properties, which is time taking and tedious job. In the present paper, a technique has been suggested to calculate maximum stresses at crack tips in a simple manner. The results can be used to establish the damage characteristic of the rock mass for improving safety and stability of existing structures and further help to improve production and productivity in mining and civil engineering constructions.

Introduction

The behaviour of rock under loading conditions is the most important parameter for any structural design within the earth or on the earth surface like tunnels, caverns and natural and man-made slopes. There are various methods and standards available to calculate the stress at the fracture tip but still there is a need to improve the methodology to calculate the stresses along the fracture plane (Verma and Singh, 2009).

Whenever a rock sample is subjected to stresses in tensile mode (mode 1), stresses in different directions *i.e.* x, y and z developed at the different points on the rock specimen along a preferential line of fracture (Cook 1965). The applied load, which induces stresses, resulted in the propagation of cracks in the rock. The developed cracks can be divided into various numbers of small distinct ellipses. These ellipses contribute to stresses in x, y, and z direction as well as shear stresses at particular point on the specimen.

The Chunar (Mirzapur district, U.P.) sandstone from lower Vindhayan Super Group has been selected for the study because it shows very less structural deformation and disturbance which are its primal characteristics. It is fine-grained,

hard, compact and massive sandstone and shows nearly isotropic and equigranular behavior under loading. It is considered to be one of the ideal rocks for parametric study. The test was performed with strain control mode(Fig.1). The rock samples were tested under uniaxial compressional loading at constant stroke rate of 0.005 mm s^{-1} .



Fig.1: Servo-controlled auto feedback stiff testing system.

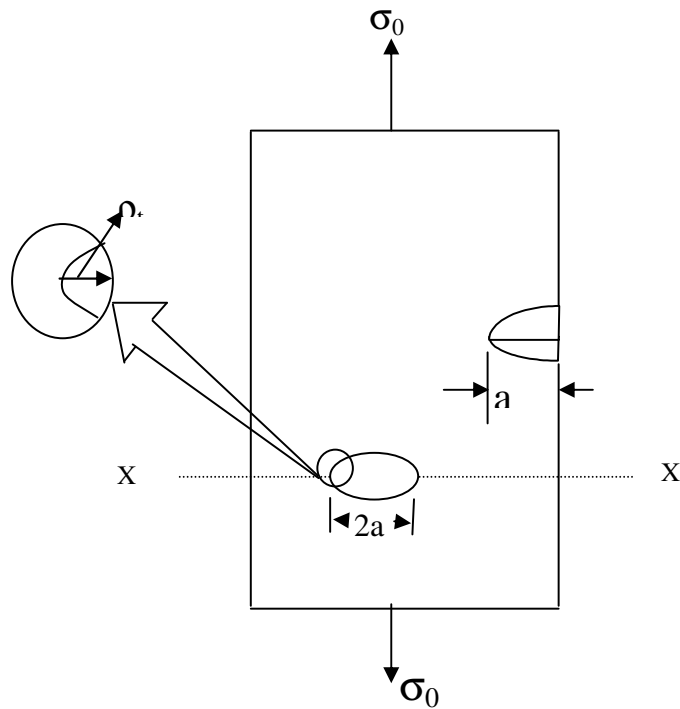


Fig. 2a: Geometry of surface and internal cracks

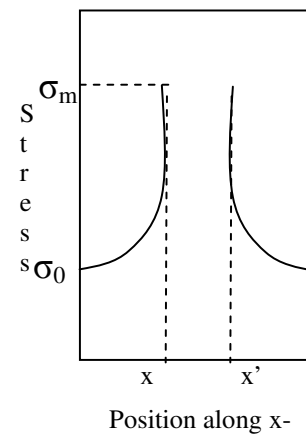


Fig. 2b: Schematic stress profile along the line x-x'

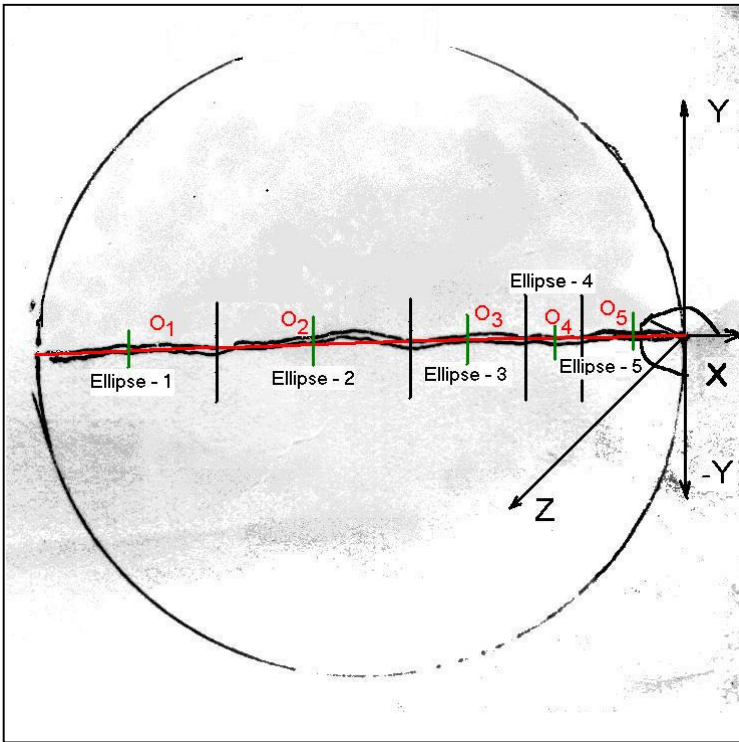


Fig. 3a: Sample No. 1 and boundaries of ellipses developed

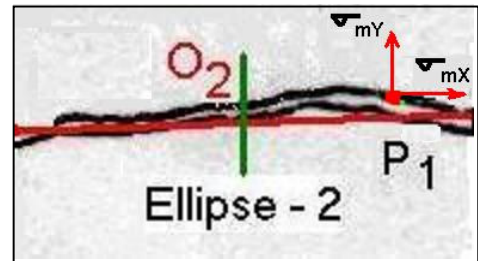


Fig. 3b: Magnified view of Ellipse 2, Sample No. 1

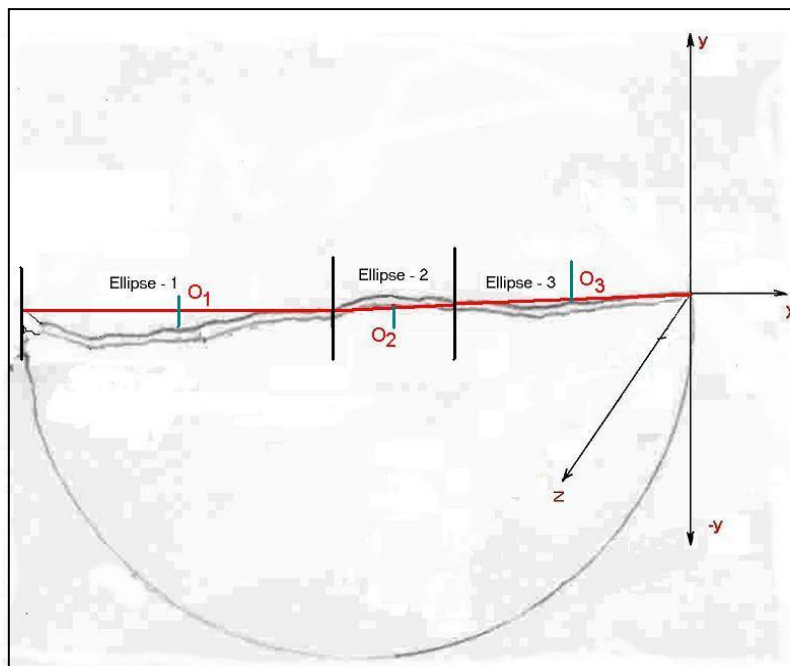


Fig. 4: Sample. 2 1 and boundaries of ellipses developed

Stress concentration

The cohesive forces that exist between different grains are functions of the fracture strength of a solid material. On this basis, it was found that one tenth's of modulus of elasticity ($E/10$) is equal to the theoretical cohesive strength of a brittle elastic material. The experimental fracture strengths of most rocks normally lie between 10 to 1000 times below its theoretical value (Goodman, 1989; Prikryl, 2001).

Griffith (1924) proposed that this discrepancy between theoretical cohesive strength and observed fracture strength could be explained by the presence of very small microscopic flaws or microcracks that always exists under normal conditions at the surface and within the interior body of the material. These flaws are a detriment to the fracture strength because an applied stress may be amplified or concentrated at the tip, the magnitude of this amplification depends on crack orientation and geometry (Whittaker and Singh, 1992).

This phenomenon has been described in Fig.2 (a-b), where a stress profile across a cross-section containing an internal crack has been drawn. As indicated by this profile, the magnitude of the localized stress diminishes with distances away from the crack tip. At positions far away from crack tip, the stress becomes equal to the nominal stress σ_0 , or the applied load divided by the specimen cross-sectional area. Due to their ability to amplify an applied stress in their locale, these flaws are sometimes called "STRESS RAISERS" (Rooke and Tweed, 1973; Goodman, 1989).

Assuming that a crack has an elliptical shape (or is circular) and oriented perpendicular to the applied stress, the maximum stress at the crack tip, σ_m is equal to

$$\sigma_m = \sigma_0 [1+2(a/\rho_t)^{1/2}] \quad (1)$$

where,

σ_0 = applied nominal stress,

a = Half of the crack length,

ρ_t = Radius of curvature of the tip, and

σ_m = magnitude of the maximum stress at the crack tip.

For a relatively long microcrack that has a small radius of curvature at the tip of the fracture, the factor $(a/\rho_t)^{1/2}$ may be very large under these circumstances and equation - 1 may be re written as

$$\sigma_m = 2 \sigma_0 (a/\rho_t)^{1/2} \quad (2)$$

The ratio σ_m/σ_0 is denoted by K_t and is called 'stress concentration factor'

$$K_t = \sigma_m / \sigma_o = 2 (a / \rho_t)^{1/2} \quad (3)$$

The factor K_t is a measure of degree to which an external stress is amplified at the tip of the crack. This factor is important to study for brittle material because the effect of stress raiser is more significant in brittle as compared to ductile material. Plastic deformation develops when the maximum stress exceeds the yield strength (Whittaker and Singh, 1992). This leads to development of more uniform stress distribution in the vicinity of stress raiser and the development of a maximum stress concentration factor less than the theoretical value. Such yielding and stress redistributions don't occur to any appreciable extent around flaws and discontinuities in brittle materials.

Fracture was initiated with the application of tensile stresses and the theoretical cohesive strength is exceeded at the tip of one of the flaws. This lead to the generation of cracks and then it rapidly migrates. If no flaws were present, the fracture strength would be equal to the cohesive strength of that rock.

Calculations

Sample.1 (Fig. 3)

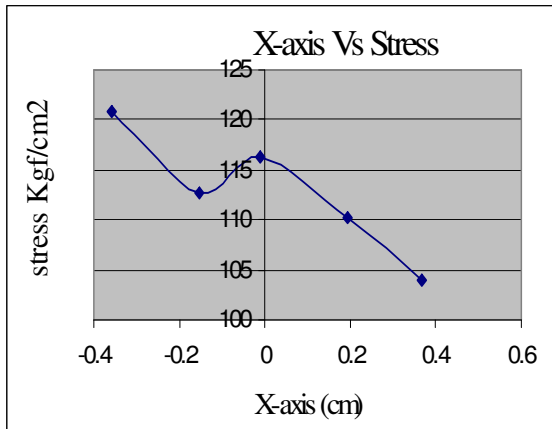
For ellipse-5

a = 0.4228 cm and b = 0.064 cm

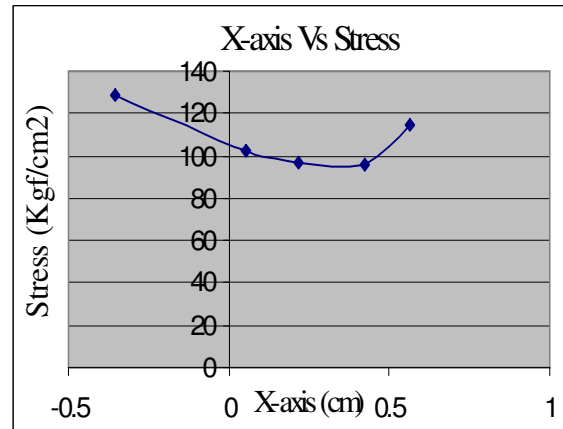
Table-1: Variation of stress at different points for ellipse 5 in sample 1.

	Point 1	Point 2	point 3	Point 4	Point 5
X (cm)	0.13	-0.043	-0.162	-0.271	-0.357
Y (cm)	0.043	0.032	0.054	0.054	0.022
ρ (cm)	0.986	-0.309	1.3033	4.4318	0.02258
K	1.30966	2.23387	2.139	1.61774	9.65437
σ_m (kgf/cm ²)	85.8207	153.253	140.166	106.008	632.641

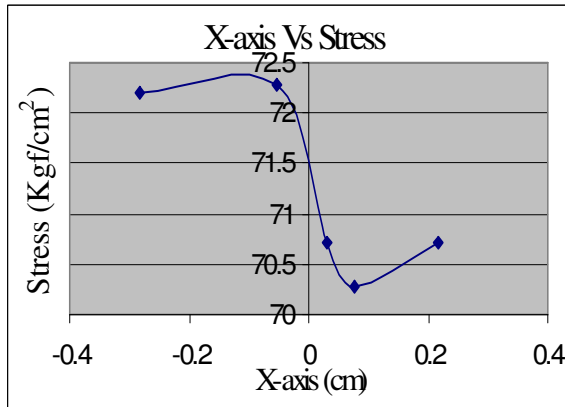
Similarly, stress values were determined at five different points for all five ellipses from the sample no. 1 and at different points of three ellipses in sample no. 2 (Fig. 3a, b)



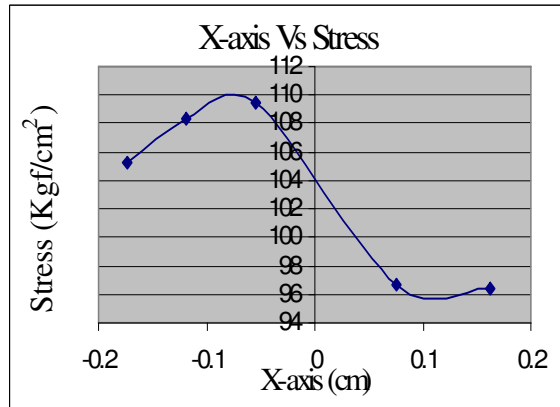
(a)



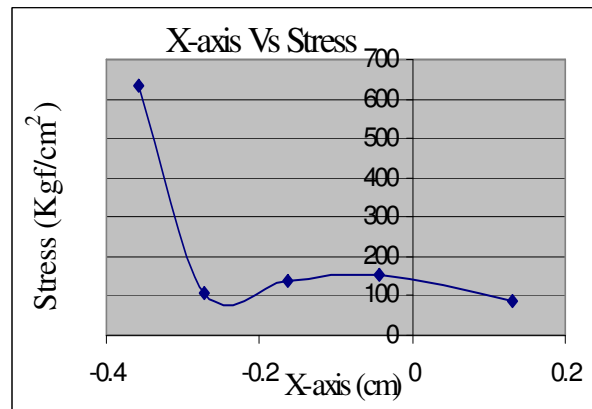
(b)



(c)

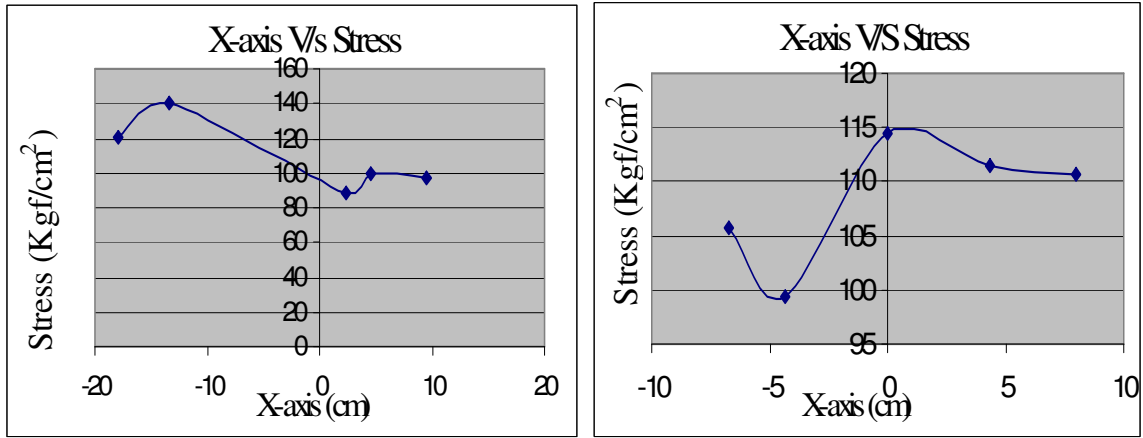


(d)



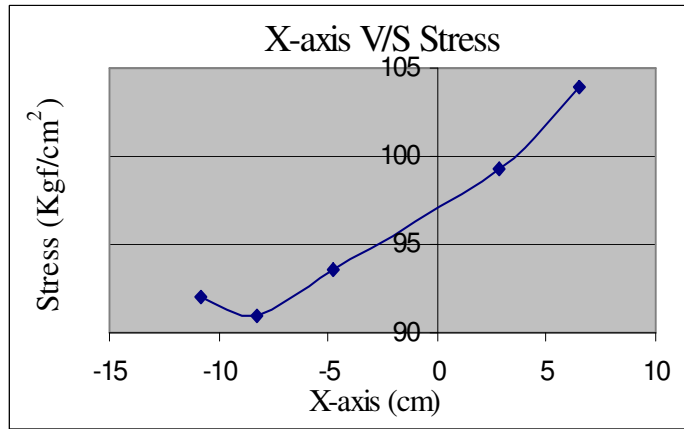
(e)

Fig. 5: X-axis V/S Stress at fracture tip for ellipse 1-5 in sample No. 1



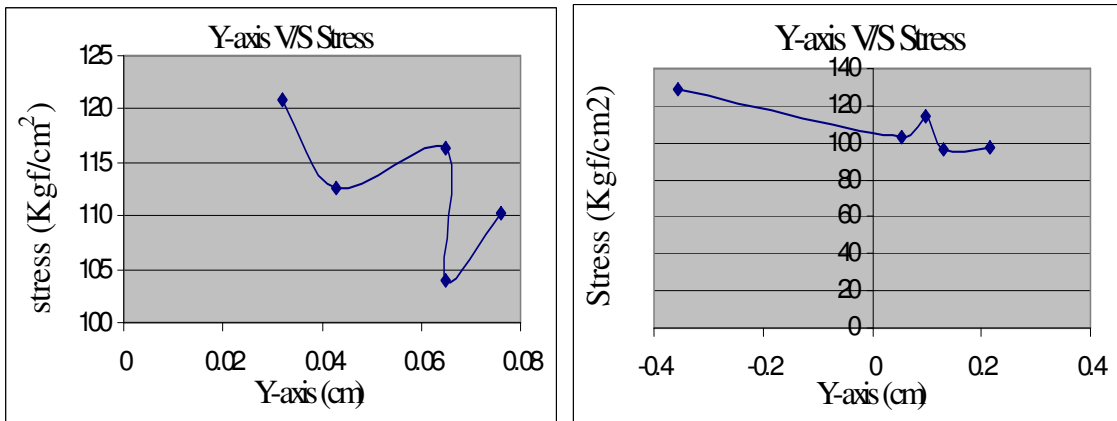
(a)

(b)



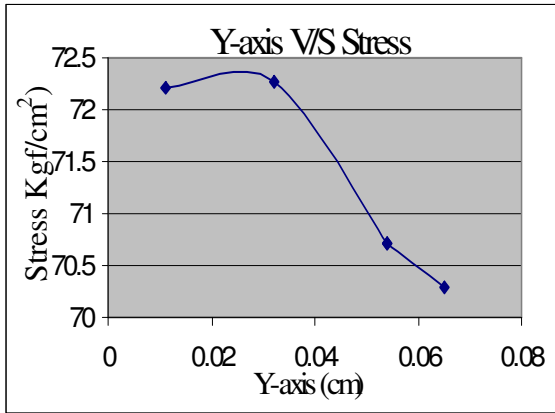
(c)

Fig. 6: X-axis V/S Stress at fracture tip for ellipse 1-3 in sample 2

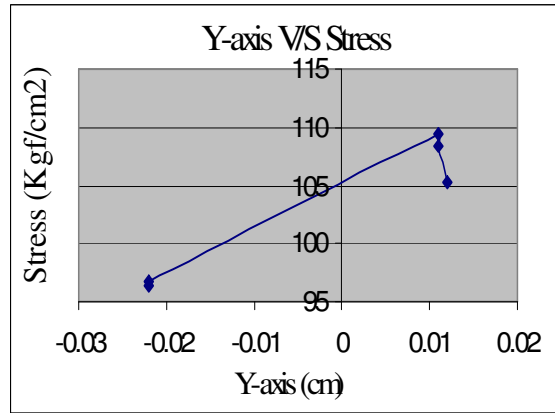


(a)

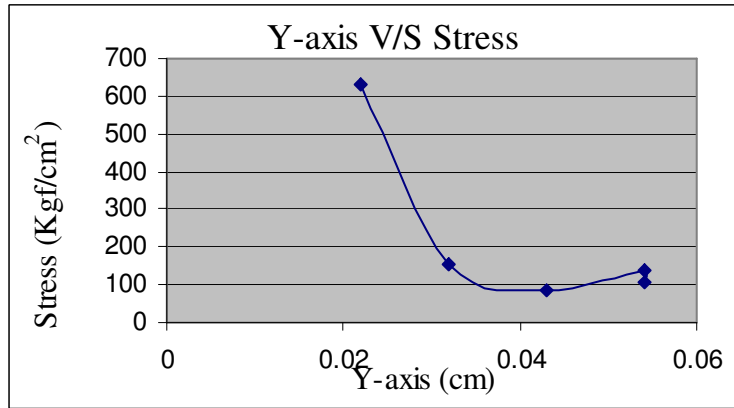
(b)



(c)

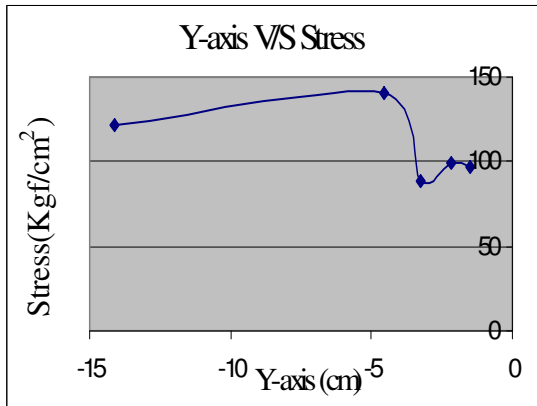


(d)

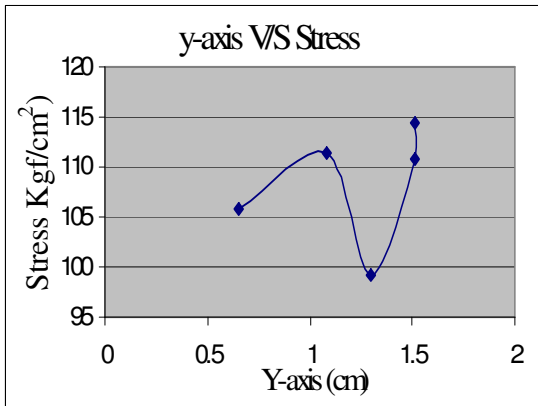


(e)

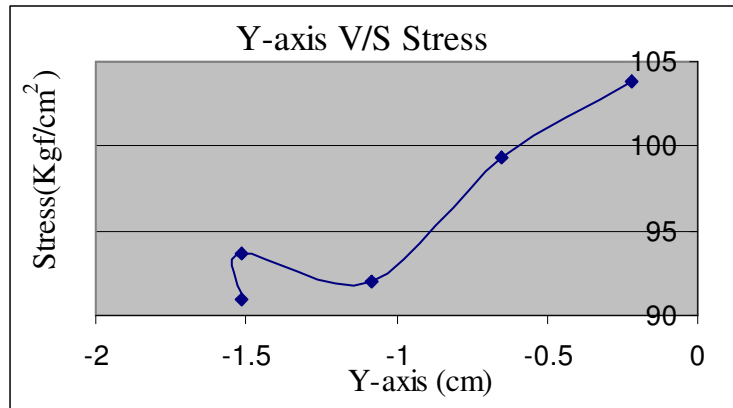
Fig.7: Y-axis V/S Stress at fracture tip for ellipse 1-5 in sample 1.



(a)



(b)



(c)

Fig. 8: Y-axis V/S at fracture tip for ellipse 1-3 in sample 2.

Results and Discussion

Maximum stress (σ_m) at the crack tips (40 points), were calculated for samples 1 and 2. Graphs were plotted for each ellipse in sample 1 and 2, showing the variation of maximum stress at tips, (with X and Y axis taking origin at the centre of the ellipse). Fig.5 a-e exhibits variation of stresses with X-axis at crack tips for 5 different ellipses of sample 1. Fig.6 a-c indicates the variation of stresses with X-axis for 3 ellipses in sample 2. Similarly, Fig.7 a-e and Fig.8 a-e shows variation of stresses with Y-axis for sample 1 and sample 2.

The sinusoidal variation of stresses is due to the presence of various micro cracks in the rock. Whenever a crack initiates, it encounters plane of weaknesses or micro cracks, the stresses become just double when the load is applied stress. When the value of this amplified stress exceeds the value of fracture toughness at a particular point on the rock, the crack propagates and extends (Hartzor and Palchik, 1997). Whenever crack forms, stress energy is used to fracture the rock and its value drops down which is called stress releaser. Afterwards, as load increases, compaction in the rock increases due to collapse, and then compaction of structure and stress concentration starts regaining its value unless and until another micro crack or plane of weakness appears therefore, it suddenly comes down due to release of stresses. This process continues until total fracture is developed. Similar is the case with shear stresses. The origin of various ellipses may be due to the presence of pore which is original space between grains that did not completely fill due to cementing material. This causes the development of microcracks while loading the rock. Due to increase in stress, these microcracks propagate, migrate, intersect and ultimately fail under the shear with a distinct fracture network. During the propagation of cracks in the sample, release of stress takes place along the fracture plane. It is found that the amount of stress released at the center of the crack is more than that at the corners. Therefore, according to the Griffith theory cracks with elliptical shape is formed. It is found from the above experiment that for each ellipse, stresses at the center of the crack will be least and maximum at the corners of ellipse. Hence, strain near the center will be maximum and near the corners it will be minimum.

Results indicate that, (Fig 5a-d and Fig.6a-c), the stress initially decreases to minima and then suddenly increases. Hence, the results obtained are getting consistency with the theory and verify the theoretical concept. Minimum stresses exactly at $x = 0$ (origin of ellipse) have not been observed, because the ellipses assumed are not perfectly elliptical in shape. Similarly (Fig.7 a-e and 8a-c), it can be seen that the results are also consistent. and the stress path increases first to maxima near the center of ellipse and then decreases sharply after that.

Conclusions

Stresses vary in all the three orthogonal direction in sinusoidal manner. Maximum stress is found to be for the ellipse which is near to loading side (around 700 Kgf/cm² for sample no. 1 and 160 Kgf/cm² for sample 2); while for sample 1 middle ellipse is having minimum stress.

It can be seen that for each ellipse stress is minimum on the fracture surface near the extremities of minor axis. It can also be concluded that the stress released is maximum at these points, due to which fracture plane elongates along x-axis, stretching it in elliptical shape.

Stress at the point on fracture surface near the extremities of the major axis of any ellipse is maximum. Thus, the fracture has lesser elongation along plane containing the major axis. From the above two cases, elliptical shapes of the crack can be explained with major axis as loading direction and minor axis as other direction.



Due to sinusoidal variation of stress (tensile and compressive alternatively) along the fracture surface in an ellipse, formation of number of small ellipses with crust and trough takes place at fracture surface.

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