Wavelet Analysis of Geophysical Time Series

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Abstract

Wavelet analysis has emerged as a potential tool for spectral analysis due to the time-frequency localization which makes it suitable for complex and nonstationary signals. In this paper, the strength of wavelet analysis is illustrated by three studies for (1) estimation of Hurst coefficient (2) ocean bathymetry data (3) gravity anomaly. The self affine behaviour is characterized by Hurst exponent and the application of wavelet transform is recommended for its estimation. Particularly, the bathymetry data can give important information related to the behavior of the crust and its correlation with other regions. The major thrust zones are delineated by transforming the gravity anomaly along Kiratpur-Leh-Panamik transect (~ 580 km) across NW Himalaya to wavelet domain.

Introduction

Spectral analysis of geophysical systems gives computational ease to derive important results by transforming the data into different domain. The development of wavelet transform (WT) (Morlet et al., 1982a, b; Grossmann and Morlet, 1984) offered variable resolution characteristics to resolve the spectral component and has various advantages over the conventional Fourier transform and windowed Fourier transform. In wavelet domain, significant information can be extracted simultaneously in time as well as frequency domain due to time-frequency localization property of the wavelets, which makes it suitable to study the nonstationary signals. The scaling of wavelets provides powerful methods to characterize signal structures such as fractal signals, singularities etc. Appropriate wavelet can be selected from families of different wavelets, which provides the flexibility to the transformation as per the application. Wavelet transform allows the analysis of both local as well as global features and thus, acts as a microscope in spectral analysis.

In the present paper, the application and advantages of wavelet transform are discussed using different data such as fractal signal, ocean bathymetry and gravity data. The application of wavelet transform to characterize the self-affine behaviour of a series is exploited in terms of estimation of Hurst coefficient. The enhanced resolution properties of wavelet transform are discussed in understanding the spectral behaviour of bathymetry data of western offshore, India. The use of wavelet transform in interpretation of gravity and magnetic data has started a new way to understand the geometry of causative sources of anomaly (Moreau et al., 1997; 1999). A case study of gravity data of NW Himalaya is discussed to show the role of wavelet transform method for delineation of major thrust zones.

Wavelet Transform

The wavelet transform has contributed significantly in the study of many processes/signals in almost all areas of earth science such as atmospheric turbulence, sea floor bathymetry, well log interpretation, image processing, ocean wind waves, potential field, seismological data etc. Different applications of wavelet transform have also shown its vital role while dealing the complex behaviour of real geophysical data. The continuous wavelet transform (CWT) of a function \( f(t) \) is defined as
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\[
W_{\psi,b}(a,b) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{a}} \psi^\ast \left( \frac{t-b}{a} \right) f(t) \, dt
\]  

(1)

where \( \psi^\ast \) is complex conjugate of analyzing wavelet \( \psi(t) \) which is also known as mother wavelet or kernel wavelet, \( a \) is the dilation (scale), which is inversely proportional to frequency, \( b \) is the translation parameter. The value of \( \sqrt{a} \) is used to normalize the energy of the function at various scales (Daubechies, 1992). In CWT, the wavelet function dilates and translates over the series continuously and thus, cover different frequency components and gives better resolved spectrum. The signals having features related to different scales at different times are properly resolved using wavelet transform.

**Wavelet Analysis of Self Affine Series**

The self affine fractals which are characterized by Hurst coefficient (H) give insight into many earth processes and their modeling such as persistence analysis of the time series, seismological studies, reservoir studies and other studies related to river discharge, rainfall, sunspot number, gravity covariance model etc (Forsberg, 1987; Korvin, 1992; De Santis et al., 1997; Turcotte, 1997; Dimri, 2000; Bansal and Dimri, 2005). The methods to calculate Hurst coefficient are sensitive to the length and gap in the data (Arneodo et al., 1995; Simonsen et al., 1998; Katsev and Heureux, 2003). For calculation of H values following methods are generally used: (a) Wavelet transform; (b) Power spectrum; (c) Roughness length; (d) Semi-variogram; (e) Rescaled range analysis. The power spectrum is calculated using maximum entropy method (MEM). For roughness length, semi-variogram and rescaled range analysis, the methodology of Malinverno (1990), Oliver and Webster (1986) and Turcotte (1997) is respectively followed. Wavelet transform method is suggested in estimating H in terms of robustness and consistency after comparing the results from different methods for synthetic self-affine series (Chamoli et al., 2007).

Self-affine models/functions are generalization of fractional Brownian motion and fractional Guassian noises (Turcotte, 1997; Malamud and Turcotte, 1999). The synthetic fractional Brownian motions of different lengths are generated for H equal to 0.4 to 0.9 using successive random addition (SRA) method (Jones et al., 1996; Turcotte, 1997) (Fig.1). The Hurst coefficients are calculated using above mentioned methods. An example for calculating the H values from the fractional Brownian motion with H = 0.7 is shown in Fig.2. For this case, calculated H values by different methods for different length of the profile are shown in Table-1. From Table-1, it may be observed that wavelet transform and rescaled range methods provide the consistent H values for different length of the profile. Other cases are compared in terms of root mean square (RMS) error for H = 0.4 to 0.9 (Fig.3). From Fig.3, it can be seen that RMS error is significantly low in the case of WT method for H = 0.4 and 0.9 and for H = 0.6, 0.7 and 0.8 both WT and rescaled range technique are giving comparable low error. From this analysis, it can be inferred that wavelet transform and rescaled range methods provide comparatively good estimates of H. For large data points almost all methods provide good results, but for short data length consistent H values are found using wavelet transform and rescaled range methods. For short time series, Katsev and Heureux (2003) also pointed out the limitations of roughness length and power spectrum methods.

This study shows that wavelet transform can be used for consistent estimation of Hurst coefficient irrespective of the length of the data.
Fig. 1 - The fractional Brownian motion generated for the H values variation from 0.4 to 0.9 (after Chamoli et al., 2007).

Fig. 2: (a) Fractional Brownian motion for H = 0.7; and calculation of H values for 500 data points by using (b) wavelet transform (WT); (c) the power spectrum (PS); (d) roughness length (RL); (e) semi-variogram (SV); (f) rescaled range (RS) methods. (after Chamoli et al., 2007)
Fig. 3- The root mean square error for the different methods for $H = 0.4$ to $0.9$ (after Chamoli et al., 2007).

Table-1: The values of $H$ calculated by different methods for a fractional Brownian motion generated with $H = 0.7$. The calculated value of $H$ does not change much with the length of the profile, when computed by using wavelet transform and rescaled range methods (after Chamoli et al., 2007).

<table>
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<th>Number of data points</th>
<th>Wavelet transform method</th>
<th>MEM method</th>
<th>Roughness length method</th>
<th>Semivariogram method</th>
<th>Rescaled-range analysis</th>
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<td>0.813±0.02</td>
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</tbody>
</table>
Wavelet Analysis of Bathymetry data of western offshore, India

Wavelet analysis has been applied to bathymetry data to study the geophysical problems in understanding the characteristics of the crust, identification of anomalous regions of geodynamic importance, comparison of spectral behaviour of the topography of planets with the regions on the earth etc (Little et al., 1993; Malamud and Turcotte, 2001). Recent applications of wavelet transform (Malamud and Turcotte, 1999; 2001) suggest that it is efficient for spectral decomposition exhibiting good spatial resolution, which cannot be achieved by Fourier transform. Wavelet theory applied to seafloor bathymetry and topography data revealed the structures, which were not resolved with the raw data (Little et al., 1993; Simons et al., 1997).

The nature of crust in the Laxmi Basin is an important issue for paleographic reconstructions of western Indian Ocean. Two types of views exist for the nature of crust of Laxmi basin (Naini and Talwani, 1983; Bhattacharya et al., 1994; Miles and Roest, 1993; Singh, 1999; Krishna et al., 2006; Bansal et al., 2005). One favours it as continental while the other as oceanic. Laxmi Ridge, about 500 to 700 km off the west coast of India, is a prominent feature which divide the continental margin of western India and the Arabian Sea into two basins (Naini and Talwani, 1983). This ridge trends NW-SE and located approximately in between 14.5ºN to 19ºN (Fig. 4). Different opinions exist on nature of crust of Laxmi Basin in the previous works. Naini and Talwani (1983) favored the rifted and subsided continental crust in the Eastern Basin on the basis of thickness of the crust and absence of sea-floor spreading type magnetic anomalies. Bhattacharya et al. (1994) explained the magnetic anomalies over Eastern Basin by sea floor spreading, which indicated that Eastern Basin is oceanic. However, the Western Basin has regular sea-floor spreading magnetic anomalies (Naini and Talwani, 1983; Miles and Roest, 1993). Singh (1999) suggested the Deccan head mushrooming, which has modified the crust beneath Laxmi Basin with a huge magmatic intrusion. Krishna et al. (2006) has interpreted the nature of crust of Laxmi Basin as continental with emplaced magmatic bodies using integrated geophysical studies. Bansal et al. (2005) interpreted the revised gravity data over the Indian Ocean and concluded occurrence of continental crust below the Laxmi Ridge and Basin.

The spectral analysis of the bathymetry data along 17º12’ N latitude between the longitude ~ 60ºE and 73ºE of the western Indian Ocean has been carried out along a profile using wavelet transform to study the nature of crust in Laxmi basin (Chamoli and Dimri, 2007) (Fig. 5). The profile covers all the major features of the region including Western Basin, Laxmi Ridge, Laxmi Basin, Panikkar Ridge, continental slope and continental shelf (Fig. 5). The bathymetry data (Smith and Sandwell, 1997) is derived from Shuttle Radar Topography Mission (SRTM) 30 plus data, which is fusion of SRTM and land topography, with measured and estimated seafloor bathymetry. The bathymetry along the profile (Fig. 5) shows the prominent features of Laxmi ridge and Panikkar ridge.

The exponent in the power law relation between wavelet variance and scale has been calculated for different segments of the profile and this exponent is correlated with different features of bathymetry to deduce the nature of crust.

For analyzing the bathymetry data, we have chosen ‘Mexican hat’ wavelet, which has been generally used for analyzing topography data (Malamud and Turcotte, 1999; 2001). Mexican hat wavelet is defined by second derivative of the Gaussian probability function and mathematically given as (Daubechies, 1992):

$$\psi(x) = c \exp(-x^2/2)(1-x^2)$$  \hspace{1cm} (2)

where $c = 2/(\sqrt{3}\pi^{1/4})$. 

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where $c = 2/(\sqrt{3}\pi^{1/4})$. 

$$\psi(x)$$ is a function that is used in wavelet analysis to analyze the bathymetry data along a profile in the western Indian Ocean.
Fig. 4- The study region and bathymetry profile across Laxmi basin is shown.

Fig. 5- Bathymetry along the profile showing the positions of Laxmi ridge and Panikkar ridge.
The seafloor bathymetry data along the profile is transformed using Mexican hat wavelet (Equation 2). The wavelet coefficients are calculated for the scales \( a = 1, 2, 4, 8, 16, 32 \) which are shown in Fig. 6. The wavelet coefficients (Fig. 6) show the spectral as well as spatial behaviour. The wavelet coefficients at extremes of profile (particularly low value at small distances) have not been included in interpretation to avoid edge effects. The spectral signatures show different behaviour towards left and right of the laxmi ridge (distance \(~ 800\) km). The curves of wavelet coefficients at different scales (Fig. 6) appear to be smooth for region left of Laxmi ridge (distance \(~ 800\) km), i.e., western basin. However, the region from Laxmi ridge towards the continental shelf shows different spectral signature than the western basin, which is further studied by doing variance analysis after dividing the profile into different sections.

Fig. 6- Calculated wavelet coefficients using Mexican hat wavelet for scales (A) \( a = 1 \); (B) \( a = 2 \); (C) \( a = 4 \); (D) \( a = 8 \); (E) \( a = 16 \) and (F) \( a = 32 \) (after Chamoli and Dimri, 2007).

If the time series considered is self-affine, then wavelet variance (V) has a power law dependence on the scale parameter ‘a’ (Malamud and Turcotte, 1999) as:

\[ V \sim a^\beta \]  

(3)

The power law exponent \( \beta \) in equation (3) is equivalent to the exponent \( \beta \) found in Fourier spectral analysis in which the power law dependence between power spectral density and frequency is analyzed. The power law exponent \( \beta \) is related to fractal dimension \( D \) (Turcotte, 1997) as:

\[ D = \frac{5 - \beta}{2} \]  

(4)

The frequencies (F) corresponding to the scales are calculated using the following equation:

\[ F = \frac{F_c}{a \cdot \Delta} \]  

(5)
where ‘a’ is the scale, ‘Fc’ is the center frequency of a wavelet, ‘F’ is the frequency corresponding to the scale and ‘Δ’ is sampling period.

The plot between log of variance and log of (1/F) is analyzed in terms of exponent β for different sections of data to analyze various topographic features (Figure 7). Fig. 7 shows the bathymetry profile and sections corresponding to various features, which are individually analyzed. First, the variance analysis is carried out to the whole data and then to the different sections. The fractal dimension of bathymetry is also calculated by Equation (4).

![Graphs showing log of variance versus log of (1/F) for different sections](image)

**Fig. 7**- (a) Different sections along the profile for wavelet variance analysis. Plot of log of variance verses log of (1/F) for (b) whole data; (c) section A (Western Basin); (d) section B (Laxmi Basin); (e) section C (from Laxmi Basin upto continental shelf) and (f) section D (continental slope and continental shelf). (after Chamoli and Dimri, 2007)

Using spectral analysis of bathymetry data, it is possible to further refine the results and interpret the behaviour of crust below the various features. The value of exponent β for whole data along the profile is 2.0771 (Fig. 7b) which is close to the spectral signatures of Brownian motion (β = 2.0) and correlates well with the power law scaling of general topography of the Earth. The value of β for section A, B, C and D of profile (Fig. 7) is 1.9367, 2.838, 2.9911 and 2.8750 corresponding to Western Basin, Laxmi Basin, region from Laxmi Basin up to continental shelf and region covering continental slope and continental shelf respectively. The calculated values of fractal dimension corresponding to the values of β are 1.53, 1.1, 1.0 and 1.06 respectively. The last three values of β and fractal dimension (D) clearly show the distinct behaviour from the first value. These results can be interpreted in the light of applications of wavelet transform to identify an anomalous crust in a Pacific bathymetry profile (Little et al., 1993) and the spectral analysis of Mars topography and Antarctic ice cap (Malamud and Turcotte, 2001). The value of β and D for the studied bathymetry profile are interpreted on the basis of similar analogy of the above mentioned two case studies. The value of β and D, which correspond to Laxmi Basin, is comparatively near to β and D value for
continental slope and shelf, than the $\beta$ and D value for Western Basin. This shows deviation of nature of crust of Laxmi Basin from Western Basin, which has been confirmed to be oceanic. The spectral behaviour of crust of Laxmi Basin is near to continental shelf and slope, which shows the nature of crust of Laxmi Basin as continental.

Generally, seismic, gravity and magnetic methods are used for studying the crustal behaviour below the ridges and basins. The wavelet analysis and fractal dimension may also be a useful tool for understanding the nature of the crust. This approach of wavelet analysis can be of great importance for the cases where other data such as gravity, magnetic etc can not be easily acquired, e.g., in planetary studies. The results of this case study show that wavelet transform separates the spectral components while retaining the spatial resolution which is required in such analysis.

**Application of wavelet transform for gravity data: an example from NW Himalaya**

The continuous wavelet transform (CWT) of Bouguer gravity anomaly along ~450 km long (projected) transect from Sub-Himalaya to Karakoram fault across Indus-Tsangpo Suture Zone (ITSZ), from south to north, is carried out to study the causative sources (Fig. 8).

The detail of the data acquisition is discussed by Banerjee and Satyaprakash (2003). Fig. 9 shows the Bouguer anomaly and elevation data used subsequently. The 580 km long traverse is projected to a profile of ~450 km length due to unavoidable strike parallel observations of the survey. The Bouguer anomaly values from the Gangetic plane along the profile are added from Jin et al. (1994) to get a better constrain on the Moho geometry of the underthrusting Indian plate in Himalayan front and to avoid the immediate edge effects.

The transformation of potential field (gravity in present case) data to some auxiliary space (such as Fourier domain) gives good information about the causative sources (Spector and Grant, 1970). The continuous wavelet transform can be employed to transform the potential field data in space scale domain (Moreau et al., 1997; 1999, Fedi and Quarta, 1998) without any a priori information. Some studies in which the WT method has been applied on potential field data are aeromagnetic data of French Guiana (Moreau et al., 1999; Sailhac et al., 2000), deep tow magnetic profiles of Central Indian Ridge and Juan de Fuca Ridge (Pouliquen and Sailhac, 2003), gravity data of Nepal Himalaya (Martelet et al., 2001) and gravity and magnetic data of Bay of Bengal (Chamoli et al., 2006). The gravity data from NW Himalaya is analysed using wavelet transform method. The wavelet coefficients have been interpreted to delineate the source parameters mainly the mean depth and subsurface attitude of the geological body responsible for the observed Bouguer gravity anomaly in the complex tectonic scenario like Himalayan orogen.

If the source is homogeneous, the relation between wavelet coefficients at two levels $a$ and $a''$ for any Poisson wavelet (Moreau et al., 1997; Martelet et al., 2001) is given as

$$ W_{\psi_{a}}(b,a) = \left(\frac{a}{a'}\right)^{\gamma}\left(\frac{a''+z_0}{a+z_0}\right)^{-\beta} \times W_{\psi_{a'}}(b\frac{a''+z_0}{a+z_0},a''), $$

where second exponent $\beta$ is related to homogeneity $\alpha$ as $\beta = \alpha + 1 - \gamma$ (for gravity profiles). The value of $\gamma$ (order of the wavelet) is known which makes the calculated value of $\beta$ directly associated to the value of $\alpha$ giving the information about the shape of the source. Equation (6) represents a set of straight lines forming cone-like pattern intersecting at the center of homogeneity of the analyzed function, i.e., pointing towards
Fig. 8: (a) Tectonic map of Himalaya showing principal structural element (Modified after Gansser, 1964; Valdiya, 1980). (b) Geological map of NW Himalaya overlaid with studied gravity profile (Kiratpur-Manali-Leh-Panamik). The data points to the south and north of present profile are derived from Bouguer anomaly contours of Jin et al. (1994). The thick blue line indicates projected profile. (compiled and modified from Thakur (1992); Srikantia and Bhargava, (1998); Pandey et al. (2004) and our own observations). HFT-Himalayan Frontal Thrust, MBT-Main Boundary Thrust, MCT-Main Central Thrust, STDS-South Tibetan Detachment System, BSZ-Baralacha la Shear Zone, ZSZ-Zanskar Shear Zone, ITSZ-Indus Tsangpo Suture Zone, SSZ-Shyok Suture Zone, KRW= Kulu-Rampur window.
the location of source which is the negative dilation $a = z_0$ (mean depth of the source) (Moreau et al., 1997; 1999). Using real Poisson wavelets, the mean depth and location of causative sources of the potential field data can be calculated geometrically by the intersection of modulus maxima lines. For any modulus maxima line, the mean depth can also be estimated by spanning approximate a priori depth interval for the dilation ranges by trial and error approach and by quantifying the linear character of transformed ridge by fitting a polynomial of degree 1. The minima in misfit curve (in least square sense) corresponds to the mean depth of the source (Moreau et al., 1999). The exponent $\beta$ can be computed [using equation (6)] from the slope of best fitted line for the plot of $\log (|W| / a^\gamma)$ versus $\log (a+z_0)$ along any modulus maxima line for different values of $z_0$. Thus, geometrical properties of modulus maxima lines make it possible to find location and mean depth of source. Although this theory is for homogeneous sources, the asymptotic behaviour of the WT extends the domain of application (Moreau et al., 1999). Sailhac et al. (2000) analyzed the advantage of using complex Poisson wavelet over real Poisson wavelet. By analyzing both the real and complex wavelet coefficients, there can be improvement in confidence of interpretation.

The Bouguer gravity anomaly across NW Himalaya is analyzed using continuous wavelet transform (CWT). Before applying the CWT, the gravity anomaly is interpolated at an interval of 1.5 km (Fig. 11a) using cubic spline taking into consideration the edge correction using the data from contours of Bouguer anomaly map (Jin et al., 1994).
Then, wavelet coefficients are calculated by CWT using complex Poisson wavelet of order 1 as suggested by Sailhac et al. (2000). A number of modulus maxima lines are extracted and the prominent modulus maxima lines (marked by arrows at abscissa) (Fig. 11b), that extend to all (low and high) dilations, are considered for further analysis. The distances at which these modulus maxima lines intersect the abscissa yield the location of causative sources. The Bouguer gravity anomaly is now analyzed using real Poisson wavelet of order 3 (Fig. 11c) giving cone-like structures and modulus maxima lines. The modulus maxima lines calculated using real wavelet, which also give prominent signature using complex wavelets, are further analyzed (Fig. 11c) to obtain depth parameters.
**Fig. 11**- Wavelet analysis of the Bouguer gravity anomaly across the NW Himalaya. (a) Projected Bouguer gravity anomaly interpolated at 1.5 km using cubic spline along the Kiratpur-Manali-Leh-Panamik profile. (b) Wavelet coefficients calculated using complex Poisson wavelet of order $\gamma = 1$. Major modulus maxima lines are marked by arrows at abscissa. (c) Wavelet coefficients calculated using real Poisson wavelet of order $\gamma = 3$ showing intersections of modulus maxima lines as $D_1$ to $D_9$. The negative dilations of $D_1$ to $D_9$ correspond to the mean depth of the causative source. (d) Plot of $\log(|W| / a)$ versus $\log(a+z_0)$ along the modulus maxima line ($D_9$) with best slope of fitted straight line showing the structure as step model ($\beta = -1$). (e) Minima misfit curve (in least square sense) showing plot of misfit verses depth where minima in the curves give the mean depth as 5 km for $D_9$.

Using negative dilations. The intersection of these extended lines shows the mean depth of the source as $D_1$ to $D_9$ (Fig. 11c). To check the accuracy of these results using real wavelet, the wavelet coefficients (Fig. 11b) derived using complex Poisson wavelet of order 1 are analyzed and the results for calculation of the structure and its mean depth for $D_9$ modulus maxima lines are shown in Fig. 11 (d, e). The calculated values of mean depths are close to the values obtained using real wavelet of order 3 (Fig. 11c) and the calculated values of $\beta$ is approximately -1 suggesting step model for the studied region. Most of the intersections e.g. $D_1$ to $D_9$ correspond well to the major structural discontinuity along the profile. The mean depth of the sources of Bouguer anomaly from the major modulus maxima lines (Fig. 11b and c) are correlated with the geological structures of the Himalayan fold thrust belt along the transect (Fig. 8).

Fig. 12 presents the identified thrust zones using wavelet analysis and the geological correlation across NW Himalaya. The basement is exposed in the antiformal Kulu-Rampur window (Fig. 8) along an out-of-sequence thrust (Pandey et al., 2003; 2004), which is reflected in the wavelet analysis with $D_2$ having ~12 km depth (Fig. 14). The MCT I corresponds to shallow reflectors ($D_3$) in wavelet analysis. Baralacha la Shear Zone (BSZ) and Zanskar Shear Zone (ZSZ) in the Tethyan sediments corresponds to the $D_5$ and $D_6$ in wavelet analysis. The southern margin of Indus Tsangpo Suture Zone (ITSZ) with the Tethyan sediments is also reflected as $D_7$ in wavelet analysis whereas the contact between ITSZ and Ladakh batholith is reflected as $D_8$. The volcanics and molasses of Shyok suture zone (SSZ) and Karakoram Fault (KF) give signature as $D_9$ in wavelet analysis.

**Fig. 12**- The identified thrust zones using wavelet transform along with the geological set up.
Conclusions

Wavelet theory has come out for better handling of limitations of Fourier transform with optimum spectral decomposition of any series in wavelet domain. The wavelet method shows consistent estimates of Hurst coefficient for short and long time series. Bathymetry data of western offshore, India exhibits scaling behaviour and the fractal dimension can be estimated by wavelet variance analysis. Wavelet analysis of bathymetry profile shows different spectral behaviour for Western Basin and Laxmi Basin. It is demonstrated that the value of fractal dimension is 1.53, 1.1, 1.0 and 1.06 for Western Basin, Laxmi Basin, region from Laxmi Basin up to continental shelf and region covering continental slope and continental shelf respectively. On the basis of these values, the nature of crust of Laxmi basin is interpreted as continental. This kind of application can be important in planetary studies where other data are not available. The wavelet analysis of gravity anomaly of NW-Himalaya highlights the effectiveness of the wavelet technique in the complex tectonic scenario like Himalayan orogen. The zones $D_1$ to $D_9$ derived using wavelet analysis correspond well to the major thrust boundaries (Fig. 12). The nonstationary and complex behavior of geophysical signals can be better handled using wavelet transform as shown by the analysis of fractal, bathymetry and gravity data.

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References


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