Spatial Variability of Rock Depth using Artificial Intelligence Techniques

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Abstract

This study describes two Artificial Intelligence (AI) techniques for predicting spatial variability of rock depth in Bangalore. Reduced level of rock \(d\) at Bangalore, India is arrived from the 652 boreholes data in the area covering 220 km\(^2\). The first AI technique uses generalized regression neural network (GRNN) that are trained with suitable spread(s) to predict \(d\) at any point in Bangalore. The second technique uses Least Square Support Vector Machine (LSSVM), is a statistical learning theory which adopts a least squares linear system as a loss function instead of the quadratic program in original support vector machine (SVM). Here, LSSVM has been used as a regression technique. A comparative study between the two developed AI techniques has been presented in this paper. The results indicate that the developed GRNN model has the ability to predict \(d\) with an acceptable degree of accuracy \{Coefficient of Correlation \((r) =0.885\), and Root Mean Square Error \((RMSE) =0.021\}\. Whereas, the developed LSSVM model predicts \(d\) with an acceptable degree of accuracy \(r =0.967\), and \(RMSE=0.004\). This study also highlights the capability of LSSVM model over GRNN.

KEYWORDS: rock depth; spatial variability; artificial intelligence; least square support vector machine; artificial neural network.

Introduction

The determination of spatial variability of rock depth is an imperative task in earth science. Spatial variability of the bed/hard rock is vital for many applications in geotechnical and earthquake engineering problems such as design of deep foundations, site amplification, ground response studies, liquefaction, microzonation etc. In the literature, spatial variability of soil properties is generally studied by combining statistical analysis of site-specific data with insights from random-field theory (Yaglom, 1962; Lumb, 1975; Alonso and Krizek, 1975; Vanmarcke, 1977; Tang, 1979; Wu and Wong, 1981; Tabba and Yong, 1981; Asaoka and Grivas, 1982; Vanmarcke, 1983; Baecher, 1984; Baker, 1984; Kulati lake and Miller, 1987; Kulati lake, 1989; Kulati lake and Ghosh, 1998; Fenton, 1998; Phoon and Kulhawy, 1999; Fenton, 1999, Uzielli \textit{et al}., 2005). The science of prediction in the presence of correlation between samples is not at all well developed in the random field method. The interpretation of trends in the data from the random-field method as true trends in the mean or simply as large-scale fluctuations is a question which can only be answered by engineering judgement. Therefore, the statistical parameters, which have been used to model a random field, are generally uncertain. Geostatistics have been used for spatial variability modeling by many researchers (Delhomme, 1979; Soulie, 1983; Kulati lake, 1989; Soulie \textit{et al}., 1990; Chiasson \textit{et al}., 1995; DeGroot, 1996). However, Random field methods and geostatistics have been applied in spatial variability modeling with limited success (Juang \textit{et al}., 2001).
With an objective of predicting the spatial variability of the reduced level of the bed/hard rock \((d)\) in Bangalore, an attempt has been made to develop models based on Generalized Regression Neural Network (GRNN) and Lease Square Support Vector Machine (LSSVM). It is also aimed at comparing the performance of these developed models for the available data in Bangalore. GRNN has been used for function approximation in this study. It has been shown that, given a sufficient number of neurons, GRNN can approximate a continuous function to an arbitrary accuracy (Specht, 1991). LSSVM is firmly based on the theory of statistical learning (Suykens \textit{et al.}, 1999). Here LSSVM has been used as a regression technique. The paper has the following aims:

1. To examine the feasibility of GRNN and LSSVM model for predicting \(d\) at any point in Bangalore.
2. To make a comparative study between developed GRNN and LSSVM model.
3. To develop an equation based on LSSVM model for the prediction of \(d\) at any point in Bangalore.

**Geotechnical Data**

A large amount of geotechnical data (depth, density of the soil, fines content, Standard Penetration Test Value \((N)\), depth of ground water table, and rock depth) consisting of 652 boreholes has been collated along with index and engineering properties of subsoil layers at different locations in Bangalore (see Fig. 1). Geotechnical data were evaluated for geotechnical investigations of several major projects in Bangalore. In total, information about the 652 borelogs has been entered into the database using a GIS with ARCINFO package. The subsurface three dimensional (3D) GIS model of Bangalore has been developed with a scale of 1:20000. Fig. 1 depicts 1km\(\times\)1km grid within the corporate boundary, along with other boundaries, ring roads...
and borehole locations. It may be noted that an average of about two boreholes are available within each grid. Geotechnical data were collated from archives of Torsteel Research Foundation (India) and the Indian Institute of Science; these data were collected as part of several major projects in Bangalore during the years 1995-2003. The data in the model are on average to a depth of 30m below the ground level. The hard rock has been identified by visual observation of the cores taken at these locations. Rock depth from ground level is the difference between the ground reduced level at borehole location and reduced level of the hard rock at the same borehole location. The reduced level of the hard rock at borehole location is the difference between the ground reduced level at borehole location and depth of overburden thickness up to hard rock for the same borehole. The depth of overburden is estimated from the available borelogs.

**GRNN Model**

In a GRNN design (Fig. 2), hidden layer weights ($W_R$) are simply the transpose of input vectors from the training set. GRNN is comprised of three layers of artificial neurons (Cigizoglu and Alp, 2006). The hidden layer consists of radial basis neurons, whose transfer function is a Gaussian. A Euclidean distance is calculated between an input vector and these weights (Specht, 1991).

$$\text{dist} = |X - W_R^j|, \text{ for } j = 1, Q \quad \text{(1)}$$

![Fig. 2: Architecture of GRNN model.](image)

Where $Q=\text{number of neurons in the hidden layer}; X=\text{input vector}; \text{dist}=\text{Euclidean distance between } X \text{ and } W_R^j$. For spatial variability of $d$ of Bangalore, $X= [\text{Latitude, Longitude}]$. The calculated Euclidean distance is then rescaled by the bias, $b$:

$$b = 0.8326/s \quad \text{(2)}$$

$$n_1 = \text{dist} \times b \quad \text{(3)}$$

Where $n_1 = \text{the adjusted distance}$, and $s = \text{the spread}$. The radial basis output is then the exponential of the negatively adjusted distance having the form (Specht, 1991):
Therefore, if a neuron weight is equal to the input vector, distance between the two is 0 giving an output of 1. This type of neuron gives an output characterizing the closeness between input vectors and weight vectors. The weight matrix size is defined by the size of the training dataset, while the number of neurons is the number of input vectors. The output layer consists of neurons with a linear transfer function, which is (Specht, 1991):

\[ n_2 = W_L \times q_l \]  

(5)

Where \( W_L \) is the weight matrix in the output layer. In designing a GRNN model, it is very important to select a proper spread(s).

The main scope of this study is to implement the above methodology for prediction of d in Bangalore. For predicting d in a given space, the two input variables (latitude and longitude of borehole in Bangalore) are used for the GRNN model in this study. There are many ways of normalizing data, but the method used in this analysis is normalizing the data against their maximum values (Sincero, 2003). In carrying out the formulation, the data has been divided into two sub-sets, such as:

(a) A training dataset: This is required to construct the model. In this study, 457(70% of total data) out of the 652 d values are considered as training dataset.

(b) A testing dataset: This is required to estimate the model performance. In this study, the remaining 195(30% of total data) data is considered as testing dataset.

's' is the most important parameter in a GRNN, has been determined using the procedure described below:

(i) Begin the training of the intended GRNN by assuming a small initial s value.

(ii) Evaluate the trained GRNN using the testing data set. This is done simply by comparing the GRNN-predicted d value with the known d value for each case in the testing data set. A root mean squared error (RMSE) of all cases is calculated:

\[ \text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} e^2}{n}} \]  

(6)

Where, \( e \) is the difference between the known d value and the predicted d value, and n is the number of cases in the testing data set.

(iii) Repeat steps (i) and (ii) above, a number of times using a gradually increased ‘s’ value. With each adopted ‘s’, a GRNN is trained and used to predict d value for each case in the testing data set, and RMSE is then calculated.

(vi) Plot the RMSE against the corresponding s value. The s that yields a minimum RMSE is considered to be the optimum ‘s’.

The GRNN model has been developed by using MATLAB.
**LSSVM Model**

LSSVM models are an alternate formulation of SVM regression (Vapnik and Lerner, 1963) proposed by Suykens et al. (2002). In LSSVM, Vapnik’s ε-insensitive loss function has been replaced by a cost function which corresponds to a form of ridge regression (Suykens et al., 1999). Consider a given training set of N data points \( \{x_k, y_k\}_{k=1}^N \) with input data \( x_k \in \mathbb{R}^N \) and output \( y_k \in \mathbb{R} \) where \( \mathbb{R}^N \) is the N-dimensional vector space and \( \mathbb{R} \) is the one-dimensional vector space. The two input variables used for the LSSVM model in this study are latitude and longitude. The output of the LSSVM model is \( d \). So, in this study, \( x = [\text{Latitude}, \text{Longitude}] \) and \( y = d \). In feature space LSSVM models take the form:

\[
y(x) = w^T \phi(x) + b \quad \text{(7)}
\]

Where the nonlinear mapping \( \phi(\cdot) \) maps the input data into a higher dimensional feature space; \( w \in \mathbb{R}^N; b \in \mathbb{R} \); \( w \) = an adjustable weight vector; \( b \) = the scalar threshold. In LSSVM for function estimation the following optimization problem is formulated (Smola, 1998):

Minimize: \[
\frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{k=1}^N e_k^2
\]

Subjected to: \[
y(x) = w^T \phi(x_k) + b + e_k \quad , k=1,…,N. \quad \text{(8)}
\]

The following equation for \( d \) prediction has been obtained by solving the above optimization problem (Vapnik, 1998; Smola and Scholkopf, 1998).

\[
d = y(x) = \sum_{k=1}^N \alpha_k K(x, x_k) + b \quad \text{(9)}
\]

The radial basis function has been used in this analysis. The radial basis function is given by:

\[
K(x_k, x_l) = \exp \left\{ -\frac{(x_k - x_l) (x_k - x_l)^T}{2\sigma^2} \right\} \quad , k,l=1,…,N \quad \text{(10)}
\]

Where \( \sigma \) is the width of radial basis function.

The plausibility of the above proposed models is evaluated using \( d \) in the subsurface of Bangalore. In LSVM model, the same training dataset, testing dataset, and normalization technique have been used as used in SVM model. The value of \( \gamma \) and width of radial basis function (\( \sigma \)) value has been chosen by trial and error approach. In the present study, training and testing of LSSVM have been carried out by using MATLAB.
Results and Discussion

Coefficient of correlation (r) has been used as main criteria to examine the performance of developed model. Fig. 3 shows the plot between s and RMSE for GRNN model. From Fig. 3, it is clear that 0.021 is the optimum ‘s’. The performance of training and testing data has been determined by using the value (s=0.021) of optimum ‘s’. Fig. 4 shows the performance of GRNN model. Fig. 4 also confirms that the developed GRNN model has capability to predict d at any point in Bangalore. In case of LSSVM model, the design value γ and σ is 50 and 0.1 respectively. Fig. 5 shows the performance of LSSVM model. Both GRNN and LSSVM have better performance in the training phase than in the testing phase.
The loss of performance with respect to the testing set addresses a machine’s susceptibility to overtraining. There is a very marginal reduction in performance on the testing dataset (i.e., there is a difference between machine performance on training and testing) for the GRNN as well as LSSVM model.

![Performance of LSSVM model.](image)

**Fig. 5:** Performance of LSSVM model.

This relatively small decline of performance of the LSSVM over GRNN model indicates its ability to avoid overtraining, and hence it can be expected to generalize better than GRNN. The following equation can (by putting $\sigma = 0.1$ and $b = -0.075$ 7 in equation-9) be developed based on LSSVM for the prediction of $d$ at any point in Bangalore.

$$d = \sum_{k=1}^{457} \alpha_k \exp \left\{ -\frac{(x_i - x)(x_i - x)^T}{0.02} \right\} - 0.0757 \quad \text{................. (11)}$$

Where, $x_i$ is the input of training dataset, $x$ is input of the data whose $d$ value has to be determined and $T$ is transpose. Fig. 6 shows the value of $\alpha$.

In order to compare between the developed GRNN and LSSVM model, five points have been chosen randomly from known reduced level of rock values of 652 points in the subsurface model of Bangalore. The predicted values of these points are shown in Table-1.
Fig. 6: Values of $\alpha$.

<table>
<thead>
<tr>
<th>Bore hole no.</th>
<th>Longitude (degree)</th>
<th>Latitude (degree)</th>
<th>Actual reduced level of the rock (m)</th>
<th>Predicted reduced level of rock (m) by GRNN</th>
<th>Predicted reduced level of rock (m) by LSSVM</th>
</tr>
</thead>
<tbody>
<tr>
<td>71-2</td>
<td>77.5765</td>
<td>12.9448</td>
<td>885.20</td>
<td>888.40</td>
<td>886.80</td>
</tr>
<tr>
<td>53-6</td>
<td>77.6237</td>
<td>12.9447</td>
<td>884.46</td>
<td>880.46</td>
<td>883.95</td>
</tr>
<tr>
<td>725-39B</td>
<td>77.6641</td>
<td>12.9924</td>
<td>893.50</td>
<td>897.50</td>
<td>892.52</td>
</tr>
<tr>
<td>51-9</td>
<td>77.5874</td>
<td>12.9331</td>
<td>896.60</td>
<td>898.60</td>
<td>897.56</td>
</tr>
<tr>
<td>87-4</td>
<td>77.5368</td>
<td>13.0293</td>
<td>900</td>
<td>897.45</td>
<td>901.35</td>
</tr>
</tbody>
</table>
Conclusion

This paper describes two artificial intelligence techniques (GRNN and LSSVM) for the determination of spatial variability of rock depth in Bangalore. In the development of the model discussed here, significant effort is required to build the machine architecture. However, once developed and trained, the transpired models performed the simulations in a small fraction of the time required by the physically based model. A comparative study between the developed GRNN and LSSVM model shows that LSSVM
is better than GRNN. Scientists can use the developed equation based on LSSVM model for the prediction of reduced level of rock at any point in Bangalore. This paper gives a powerful model based on LSSVM for the prediction of spatial variability of reduced level of rock in Bangalore.

**References**


